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Research Paper

Psychomechanics and the Trio of Space, Time and Consciousness: The Principle of Minimum Effort

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ABSTRACT

In the realm of classical physics, there are several phenomena whose precise understanding requires the role of consciousness of the observer/instrument. In such cases all the three in the trio of space, time and consciousness need to be considered in the theory on the same footing rather than neglecting the role of consciousness under some simplifying assumptions. An easy way to incorporate consciousness in the theory is by way of introducing a meditation variable, μ, and treating the same at par with the time variable. It may be mentioned that it is the consciousness that creates the event-based space, time and geometry in the outer world out of the experience-based space, time and geometry of the mental world. In fact, at gross level the role of 'mind' (an attribute of consciousness) is now translated into the variable μ which, in some sense, also accounts for the attention and/or concentration of an individual. Some possible factors are explored on which the intensity of meditation can depend. An energy-like function for conscious systems, the so-called 'mindset function' $\mathcal{M}$, is introduced. In general, $\mathcal{M}$ is a function of experience-based space and time or of event-based space and time including μ. In analogy with the action integral of classical mechanics, an 'effort integral' is constructed here perhaps for the first time and the extremeness of this integral is investigated using the tools of calculus of variation. The viability of the resultant psycho-dynamical equations is demonstrated by solving them for some particular situations.

Keywords: *Consciousness energy, meditation variable, experience-based and event-based space and time intervals, effort integral.*

With regard to the absolute reality in Nature recall the three world concept of Karl Popper and Eccles (Popper and Eccles, 1981). The three worlds considered here are: world-1, which really exists; world-2, which can be perceived, and world-3, which can be modeled by (objective) scientists. Ideally speaking, the world-1 is purely a matter of realization and the world-2, involving the individual-based process of perception or the 'internalization of the exterior', can be explored in terms of space, time and consciousness or the variants of the latter, like mediation, attention, concentration, etc., can also be used. The world-3 is the game of space-time mediated models and the associated interactions, of course for a conscious observer whose consciousness is often ignored at every stage of modeling

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including the stage of developing the mathematical aspects. For example, recall the statistical methods that are used to eliminate the individual-based errors in the direct measurement processes in the classical context or the probabilistic methods that are employed in the quantum context.

In the same way as any discrepancy between the measured truth about the world-3 and the perceived truth about the world-2 sets the limits on the modeling processes of objective science, the discrepancies noticed between the perceived truth about the world-2 and the realized truth even by an individual about the world-1, set the limits on the tools of psycho-science (Kaushal, 2011), i.e. of psycho-anatomy and /or psychophysiology of the 'mind' itself. As a matter of fact the description of any phenomenon pertaining to world-2 in general would require an account of all the three ─space, time and consciousness, on the same footing. The consciousness in the Western schools of thought is considered as an attribute of just 'mind'. But in reality it is not so (Kaushal, 2011; Kaushal, 2015), it has rather further details at subtle level in Vedic wisdom. Here, we restrict ourselves from going into these details about consciousness but introduce for it a new variable, the so-called 'meditation variable' μ, in the theory. As a matter of fact μ will account for the entire spectrum of consciousness covering all 'islands' of intangible world including the soul ─assumed to be the source of consciousness in an individual. As discussed in Vedic literature, some of these islands are: mind (M) ─as the faculty of emotions, intellect (I) ─as the faculty of decision, self-sense or ego (E) ─as the faculty of memory, etc.

In the next section we introduce notations and terminology that will be used later in the text. In Sect.3, we introduce, perhaps for the first time, the concept of experience-based and event-based space and time durations. Note that the latter category of space time forms the basis for the description of World-3. In analogy with the action integral of classical mechanics (see, Chapter 2 of (Goldstein, 1981)) we construct an 'effort integral' in Sect.4. Then, after using the tools of calculus of variation, psycho-dynamical equations for the evolution of the 'mindset function' $\mathcal{M}$, are derived in this section. A guide for the use of these equations is suggested in Sect.5. As the handling of these psycho-dynamical equations becomes difficult even for simple cases a particular class of the 'potential function' $V(q,t,\mu)$ is investigated in Appendix A to get a feel for the solutions. With a view to widening the scope of applications of some of these results, additional remarks are made on experience- and event-based space time intervals in Appendix B. Finally, concluding remarks are made in Sect. 6.

Notations and Terminology

(i)Note that for a conscious person the variable μ is never zero but it can be infinity for a liberated individual. We shall consider μ at par with the time variable t in spite that $t=t(\mu)$ and the space variable, q is now, $q=q(t,\mu)$, in implicit form. As a matter of fact in this experience-based world, the 'space variable' q, is no more a directional quantity (as is the case in the event-based world), rather it is a distribution in the space of t and μ. This is what one experiences during the initial stages of meditation. In some sense we want to restore 'matter over mind' or 'bottom-up' approach (see, Chapter 4 of (Kaushal, 2015)) to account for the psyche of an individual.

(ii) *Consciousness energy and the mindset function*: In analogy with the energy functions like Lagrangian and Hamiltonian of classical mechanics, we introduce here consciousness-energy (abbreviated here as 'c-energy' or termed as 'cenergy') function, $\mathcal{M}$, also called as 'mindset function'. As a matter of fact the state of mind at any stage of meditation is characterized by this 'mindset function' $\mathcal{M}$. Its evolution with reference to a particular task can be considered with respect to space (q), time (t) and/or meditation variable μ. The latter is a measure of consciousness. Note that in general $\mathcal{M}$, in given circumstances, is not only a function of q, t and μ , viz, $\mathcal{M} = \mathcal{M}$(q(t,μ), t(μ), μ) but also of the respective derivatives of q and t, namely of $\dot{q} = (\partial q / \partial t)$, $\overline{q} = (\partial q / \partial \mu)$ and $\bar{t} = (\partial t / \partial \mu)$. Thus, $\mathcal{M} = \mathcal{M}\ (q, \dot{q}, \overline{q}, t, \bar{t}, \mu)$. Unless and otherwise mentioned, the dependence of q and t on their arguments remains only implicit. Further, note that at finer level $\mathcal{M}$ can also be a function of the variables m, i and e responsible respectively for the roles of essences of life M, I, E in place of μ (Kaushal, 2011). Here $\overline{q}$ and $\bar{t}$, in some sense, belong to the experience-based world of space time, and q represents a compactified version of space as far as the concept of physical dimensions is concerned. We shall avoid such possibilities in this work.

(*iii*) *Thinking fast and slow: Gross and subtle components of* $\mathcal{M}$*:* With reference to the psychology of an individual, the Nobel Laureate Daniel Kahneman, in his bestseller book "*Thinking, Fast and Slow*", emphasized (Kahneman, 2011) the two aspects of human thinking process, namely fast and slow. In fact, the two selves introduced by Kahneman through 'system 1' and 'system 2' in his theory of economic science, respectively are in accordance with the gross component $\mathcal{M}_g$ and subtle component $\mathcal{M}_s$ of Vedic literature. (For details, see Appendix B in the 2nd ed. of (Kaushal, 2015) and Sect. 5.4 in (Kaushal, 2017a)). In view of these considerations it is not difficult to consider the mindset function $\mathcal{M}$ as consisting of two parts, namely conscious or active (here referred to as "dynamic") part that corresponds to the gross component $\mathcal{M}_g$, and sub-conscious or passive (here referred to as "static") part that correspond to the subtle component $\mathcal{M}_s$. Thus, it is reasonable to write the function $\mathcal{M}$ as the sum of dynamic and static parts in analogy with the 'kinetic' and 'potential energy' terms in the Hamiltonian function of classical mechanics (Goldstein, 2011).

Further note that the dynamic part in $\mathcal{M}$, primarily depending on $\dot{q}, \overline{q}, \bar{t}$, is positive definite since an individual always makes an effort and also has tendency to survive in this world through his active mind. The passive or static part, on the other hand, defines his attitude towards the world- /cosmic-order in general. It could be negative to convert $\mathcal{M}$ into a Lagrangian function of classical mechanics or could be positive to retain $\mathcal{M}$ as Hamiltonian (Goldstein, 1981). While such details might be useful in solving specific problems of human behavior, in either case $\mathcal{M}$, however, represents the cenergy. At times, it is the environment that forces one to take the positive or negative attitude at subtle (sub-conscious) level, otherwise everything goes well in positive spirit at the conscious level. Thus, $\dot{q}, \overline{q}, \ and\ \bar{t}$ though derived from q and t, but can be considered as independent variables in the spirit of canonical momenta that could be derived accordingly from $\mathcal{M}$.

Note that in psycho-mechanics, like in classical mechanics (Goldstein, 1981), the conserved quantities can be defined and traced with respect to both μ and/or t. As far as the physics relevance of these conserved quantities is concerned, clearly, those with respect to t will correspond to the physical world-3, those with respect to μ will belong to world-1, and those with respect to both μ and t will correspond to world-2. For example, many behavioral facts are found (see, Chapter 8 of (Kaushal, 2015)) to be universal, particularly with reference to the personality of an individual, whereas some facts are realized universally but hard to express them through limited means of speech, letter or actions. We refrain ourselves from going into these details here but refer to our earlier works (Kaushal, 2011; Kaushal, 2015; Kaushal, 2017a).

(iv) Meditation and the intensity of meditation: When a person sits in meditation, the depth or the degree of achieving a focused attention on a thought is characterized by μ. In some sense, μ is a collective representation of the parameters responsible for the roles of inner essences of life, namely mind, intellect and ego, in a decision making process towards the success of a task. Note that μ can never be zero for an alive person but it can be infinite for a super enlightened person, i.e. for the yogi-category of persons (see, Glossary of Words in (Kaushal, 2017)). For a common man, however, μ lies between zero and infinity. As a first step towards the quantification, our common experience reveals that the intensity of meditation is (a) inversely proportional to the frequency f of life-breaths, (b) directly (inversely) proportional to the number of positive (negative) thoughts N_+ (N_-), (c) inversely proportional to the time duration τ assigned to complete the task, and (d) directly proportional to the capacity C of controlling the mind by an individual from it's running into the sense-objects. Symbolically, one can write

$$\text{Intensity of meditation or of focused attention } (\mu) = \frac{\kappa C N_+}{\tau f N_-}, \qquad (1)$$

where κ is a constant of proportionality. About the formula (1), the following remarks are in order:

1. The proportionality constant κ in general is a function of circumstances, σ or of the mental set-up affected by the environment.
2. The linear dependence of μ on C is rather an ideal and simplified situation; otherwise C in general could be replaced by a complicated function of C, say f(C). This is mainly because that there are five categories of stimulants which can affect the turbulence of mind in different proportion and style. As a matter of fact, in this contemplative state the senses become so inhibited that the individual does not hear, smell, taste or see, nor does one experience touch. During this single-mindedness, the ideas revolve around the object of meditation. As a result, sensory stimuli are one excluded, the mind ceases to imagine and the time slows down (Vrobel, 2011).
3. Note that the life-breathing frequency f and the assigned time duration τ, themselves have mutual dependence in the sense that if the assigned time is less, then for a common man f increases with reference to the conscious mind (or the gross component of M). This kind of dependence of μ on τ, on the other hand, also supports the "incomprehensibility principle" noticed by Fraser (Fraser, 2013) according to which the intensity of one's consciousness (attention or meditation) and the duration for which it is applied seriously affect the outcome of observation, i.e. they are inversely proportional.

EXPERIENCE-BASED AND EVENT-BASED SPACE AND TIME INTERVALS

In formula (1) note that τ denotes the real time duration and such a dependence of μ on τ was noticed by Fraser during his assignment as the editor of CERN courier. In reality the time duration may arise with reference to both mental (virtual) and physical (real) worlds. The former actually deals with the experience-based time and the latter with the event-based time. In order to have an experience, naturally the experiencer has to have an element of consciousness and the latter in some sense is the creator of time. Such a thing in fact is missing in the event-based time. Further, in the event-based case there is some sort of 'internalization of the exterior' where as in the experience-based case there is 'externalization of the interior' and here one talks of the 'subjective' time. In fact, whatever is true for the time durations, the same is true for the space intervals. As a matter of fact a common experience reveals that the experience-based space and time intervals (durations) are contracted (i.e. they fly like anything) if the experiences are pleasant or appealing and they are elongated if the experiences are otherwise (unpleasant or not favorable). Susie Vrobel, while investigating the fractal time, has recently advanced several new concepts in this connection, particularly about the texture of time in Nature (Vrobel, 2011).

What Vrobel emphasizes is that even in the event-based time duration there could be an experience-based part which is either not detected by the physical instrument or accounted through the so-called personal errors in the measurement. To this effect, Vrobel introduces the concepts of 'length' and 'depth' of time in the given interval, which, respectively correspond to 'the duration' and 'the now' and accordingly characterize the 'succession' and 'simultaneity' of events during observation. It is 'the now'-part of the time interval that is subjective and the same can be identified only by a conscious observer and also not unique for each observer. On the other hand, in the experience-based time interval the whole interval is a subjective time– there could be a slow or fast flow of time, or the time is dragged or flies of, etc. For example, one notices the flying of time during suitable enjoyment or dragging of the time during the adverse conditions.

In what follows, we list some phenomena and common experiences involving consciousness and belonging mainly to the world-2:

(i) When one crosses an unusually crowded road at zebra mark or when one drives through a crowded lane without divider, all three space, time and consciousness play comparable roles in decisions.

(ii) As mentioned before, in the observation by Fraser (i.e. the product of intensity of attention and the time span during which the attention is applied is a constant) it is the event-based time duration that appears (cf. eq.(1)).

(iii) Time sharpens the mind: Akin to the incomprehensibility principle of Fraser (Fraser, 2013), this is another common-man experience. In fact, as the time of delivering a speech or lecture or taking an examination approaches, the inner faculties of learning (including that of memory and decisions) become more and more efficient or sharp.

A simple model for the consciousness evolution with event-based time duration can perhaps offer an explanation to this phenomenon. To this effect, we assume that (Kaushal, 2017) at any time t the rate of increase of μ is given by

$$(d\mu/dt)\ \alpha\ \frac{1}{t_0-t} \quad or \quad \frac{d\mu}{dt}=\frac{\lambda}{t_0-t} \quad , \tag{2}$$

where λ is the constant of proportionality and t_0 is the time allotted to complete the task. The integration of (2) leads to

$$\mu(t)=\mu_0-\lambda\ln\left(1-\frac{t}{t_0}\right) \quad , \tag{3}$$

which, in the case when t<<t_0, reduces to

$$\mu(t)\simeq\mu_0+\frac{\lambda t}{t_0} \quad , \tag{4}$$

which implies a slow attention on the task at the initial stages. Such considerations can possibly suggest a link between the experience-based (Δt_{ex}) and event-based (Δt_{ev}) time durations (cf. phenomenon (v) below).

(iv) Chevreul hand-held pendulum: About 180 years ago, French Chemist Chevreul tried to demonstrate that the role of consciousness can be noticed in the hand-held pendulum experiment when the variations in time period are analyzed in different situations. Recently, an experiment to this effect is proposed (Kaushal, 2016) by the author.

(v) Weber-Fechner law: This law quantifies the perception of change in a given stimulus. The perception process as such has roots in consciousness or in the intensity of attention. The law states that the perceived intensity (loudness/brightness) (I_p) is proportional to the logarithm of actual intensity (I_a), viz.,

$$I_P \,\alpha \ln I_a \quad or \quad I_a = e^{I_P/k} \quad , \tag{5}$$

where k is the constant of proportionality, it however depends on the type of the perceived sense-object namely touch, taste, seeing, hearing or smell and also on consciousness or on the strength of meditation µ. Without any loss of generality one can as well use the Weber-Fechner law (5) to connect the experience-based (perceived) time interval (Δt_{ex}) and the event-based (nascent or actual) time interval (Δt_{ev}) to give a relation of the form

$$\Delta t_{ev} = A \exp\left(\Delta t_{ex} / k \right). \tag{6}$$

Note the possibility of understanding Weber-Fechner law (5) or the relation (6) in the simple model (3), of course after some dimensional considerations. As a matter of fact the relation (3) would be more appropriate when Δt_{ex} and Δt_{ev} are replaced by the corresponding time density as defined by Vrobel (Vrobel, 2011). (Note that the number of succession ('length' of time) and simultaneous ('depth' or 'the now') events in an interval Δt define the 'density' of time) . Here, *k* can be negative or positive depending respectively on the pleasant or unpleasant experiences that one notices during Δt_{ex} or when the time-flying or time-dragging situations are there, and *A* is a dimensional constant. For example, while watching a movie or in touch with a lover, time flies like anything. On the other hand, during the period of waiting for a beloved one or in touch with a hot plate, time is dragged.

(vi) Feeling of plurality of consciousness of the self or of another person: (a) A metallic ball placed between the two adjacent twisted fingers, when set in motion, gives a feeling of two balls (a case of division of the same consciousness); (b) when the other person itches your body, it gives a different, perhaps enjoyable feeling that makes you laugh, as compared to the case when you yourself itch the same part of the body (a case of interaction of two consciousness).

One can trace many such phenomena in day-to-day life that belong to world-2. Using a relation similar to eq.(6) for the space interval Δq, we discuss further viability of the Weber-Fechner law in the present context in Appendix B.

The Principle of Minimum Effort

In mathematical terms here we look for an explanation of a common experience that we all have, namely we all want "to do less and accomplish more" or each of us wants to put least effort towards the success of a task. Note that the concept of the word 'effort' here is different from that of 'action' used in physics in the sense that the former sounds an element of one's mental capability which again has roots in the individual's consciousness. Keeping in mind the preliminaries discussed in Sect.2, here we employ the tools of calculus of variation to obtain the extremeness of the functional of 'effort integral', K, defined as

$$K = \int_{t_1}^{t_2} \int_{\mu_1}^{\mu_2} \mathcal{M}\,(q,\,\dot{q},\,\bar{q},\,t,\,\bar{t},\,\mu)\,d\mu\,dt\ . \tag{7}$$

This we carry out in analogy with the 'action integral',

$$I = \int_{t_1}^{t_2} L(q,\dot{q},t)\,dt\,, \tag{8}$$

of classical mechanics (Goldstein, 1981). As mentioned before, $\mathcal{M}$ here is the measure of energy in the same way as the Lagrangian L in classical mechanics.

Next, we introduce two arbitrary parameters α (corresponding to the t-variable) and β (corresponding to the μ-variable) and write (7) as

$$K(\alpha,\beta) = \int_{t_1}^{t_2} \int_{\mu_1}^{\mu_2} \mathcal{M}\,(q(t,\mu,\alpha,\beta),\dot{q}(t,\mu,\alpha,\beta),\bar{q}(t,\mu,\alpha,\beta),t(\mu,\beta),\bar{t}(\mu,\beta),\mu)\,d\mu dt\,. \tag{9}$$

Note that the varying paths, characterized by the parameters α and β, are now in the three dimensional space of q, t, μ. The conscious Being (b) (a capsule of functional consciousness present and works in all individuals (see, Appendix A of (Kaushal, 2011)) until their death) follow such trajectories in this space between two states of mind, characterized by the point 1 and point 2 in this space. The arbitrary parameters α and β are such that they vanish at these end-points of the path. Further note that for the effort K to be an extremum, one should have

$$\frac{\partial K}{\partial \alpha}\Big|_{\alpha=0,\,\beta=0} = 0 \quad and \quad \frac{\partial K}{\partial \beta}\Big|_{\alpha=0,\,\beta=0} = 0\,. \tag{10a, b}$$

These conditions, when used in (9), will give the following pair of equations

$$\int_{t_1}^{t_2} \int_{\mu_1}^{\mu_2} \left[\frac{\partial \mathcal{M}}{\partial q}.\frac{\partial q}{\partial \alpha} + \frac{\partial \mathcal{M}}{\partial \dot{q}}.\frac{\partial \dot{q}}{\partial \alpha} + \frac{\partial \mathcal{M}}{\partial \bar{q}}.\frac{\partial \bar{q}}{\partial \alpha} \right] d\mu dt = 0 \tag{11a}$$

$$\int_{t_1}^{t_2} \int_{\mu_1}^{\mu_2} \left[\frac{\partial \mathcal{M}}{\partial q}.\frac{\partial q}{\partial \beta} + \frac{\partial \mathcal{M}}{\partial \dot{q}}.\frac{\partial \dot{q}}{\partial \beta} + \frac{\partial \mathcal{M}}{\partial \bar{q}}.\frac{\partial \bar{q}}{\partial \beta} + \frac{\partial \mathcal{M}}{\partial \bar{t}}.\frac{\partial \bar{t}}{\partial \beta} \right] d\mu\,dt = 0\,. \tag{11b}$$

Now, after noting that

$$\frac{\partial \dot{q}}{\partial \alpha} = \frac{\partial}{\partial t}\left(\frac{\partial q}{\partial \alpha}\right);\ \frac{\partial \bar{q}}{\partial \alpha} = \frac{\partial}{\partial \mu}\left(\frac{\partial q}{\partial \alpha}\right);\ \frac{\partial \dot{q}}{\partial \beta} = \frac{\partial}{\partial t}\left(\frac{\partial q}{\partial \beta}\right);\ \frac{\partial \bar{q}}{\partial \beta} = \frac{\partial}{\partial \mu}\left(\frac{\partial q}{\partial \beta}\right),\ \frac{\partial \bar{t}}{\partial \beta} = \frac{\partial}{\partial \mu}\left(\frac{\partial t}{\partial \beta}\right),$$

and using these results in (11a) and (11b), perform the integration of the corresponding terms by parts with respect to t or μ as the case may be (except for the first terms, in eqs.(11a) and (11b)). For example, the second term in (11a) yields (Goldstein, 1981)

$$\int_{t_1}^{t_2} \int_{\mu_1}^{\mu_2} \frac{\partial \mathcal{M}}{\partial \dot{q}}.\frac{\partial}{\partial t}\left(\frac{\partial q}{\partial \alpha}\right) d\mu\,dt ==$$

$$= \int_{\mu_1}^{\mu_2} \left[\frac{\partial \mathcal{M}}{\partial q}.\frac{\partial q}{\partial \alpha} \right]_{\substack{\alpha=0\\ \beta=0}} d\mu - \int_{\mu_1}^{\mu_2} \int_{t_1}^{t_2} dt \left[\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{M}}{\partial q}\right)\frac{\partial q}{\partial \alpha} \right] d\mu$$

$$= 0\ -\ \int_{\mu_1}^{\mu_2} \int_{t_1}^{t_2} \frac{\partial}{\partial t}\left(\frac{\partial \mathcal{M}}{\partial q}\right)\frac{\partial q}{\partial \alpha}\,dt\,d\mu.$$

Performing similar operations on each term in eqs.(11a) and (11b) (except for the first terms) and then the subsequent use of conditions (10) yields

$$\int_{t_1}^{t_2}\int_{\mu_1}^{\mu_2}\left[\frac{\partial}{\partial t}\cdot\left(\frac{\partial\mathcal{M}}{\partial\dot{q}}\right)+\frac{\partial}{\partial\mu}\left(\frac{\partial\mathcal{M}}{\partial\overline{q}}\right)-\frac{\partial\mathcal{M}}{\partial q}\right]\left(\frac{\partial q}{\partial\alpha}\right)_{\alpha=\beta=0} dt\,d\mu=0 \tag{12a}$$

$$\int_{t_1}^{t_2}\int_{\mu_1}^{\mu_2}\left\{\left[\frac{\partial}{\partial t}\cdot\left(\frac{\partial\mathcal{M}}{\partial\dot{q}}\right)+\frac{\partial}{\partial\mu}\left(\frac{\partial M}{\partial\overline{q}}\right)-\frac{\partial\mathcal{M}}{\partial q}\right]\left(\frac{\partial q}{\partial\beta}\right)_{\alpha=\beta=0}+\left[\frac{\partial}{\partial\mu}\cdot\left(\frac{\partial\mathcal{M}}{\partial\bar{t}}\right)-\frac{\partial\mathcal{M}}{\partial t}\right]\left(\frac{\partial t}{\partial\beta}\right)_{\beta=0}\right\}d\mu\,dt=0 \tag{12b}$$

Next, we make use of the "fundamental lemma" of calculus of variation (Goldstein, 1981) after writing the Taylor expansions of the functions q(t,μ,α,β) and t(μ,β) as

$$q(t,\mu,\alpha,\beta)\simeq q(t,\mu,0,0)+\alpha\eta_1(t,\mu)+\beta\eta_2(t,\mu)\quad,$$
$$t(\mu,\beta)\simeq t(\mu,0)+\beta\zeta(t)\quad,$$

where the functions η_1, η_2 *and* ζ are the arbitrary and independent functions of their arguments. They vanish at the end-points of the paths which essentially vary for a particular parametric family. Then the lemma, in the present case, states that (Goldstein, 1981)

$$\iint M(x,y)\eta_1(x,y)\,dxdy=0$$
$$\iint M(x,y)\eta_2(x,y)\,dxdy+\iint N(x,y)\zeta(y)\,dxdy=0$$

and hence implies M=0, N=0. This will result into a pair of equations (analogous to Lagrange equations of motion), viz.,

$$\frac{\partial}{\partial t}\left(\frac{\partial\mathcal{M}}{\partial\dot{q}}\right)+\frac{\partial}{\partial\mu}\left(\frac{\partial\mathcal{M}}{\partial\overline{q}}\right)-\frac{\partial\mathcal{M}}{\partial q}=0 \tag{13a}$$

$$\frac{\partial}{\partial\mu}\left(\frac{\partial\mathcal{M}}{\partial\bar{t}}\right)-\frac{\partial\mathcal{M}}{\partial t}=0. \tag{13b}$$

Eq.(13a) describes the evolution of the cenergy function with respect to all the three ─space, time and consciousness whereas eq.(13b) describes the evolution with respect to time and consciousness. In general both the equations need to be explored for the complete solution of the problem, but it depends on the nature of the phenomenon under study to use (13a), (13b) or both.

Cyclic Coordinates: Next, we analyze the case of 'cyclic coordinates' in analogy with the classical mechanics (Goldstein, 1981). In fact, in case when $\mathcal{M}$ does not depend explicitly on q and/or t then one can derive some conserved quantities for the system which , in turn, will help in solving the problem.

Case (i): When $\mathcal{M}$ does not depend explicitly on t, then (∂M/∂t)=0 and eq.(13b) gives

$$\frac{\partial}{\partial\mu}\left(\frac{\partial\mathcal{M}}{\partial\bar{t}}\right)=0\qquad or\qquad\left(\frac{\partial\mathcal{M}}{\partial\bar{t}}\right)\equiv C_\mu=\text{constant (with respect to }\mu\text{)}. \tag{14}$$

We term C_μ here as the conjugate cenergy of the system, which in some sense defines the asymptotic limit of one's meditating capacity. This perhaps is the constant that appears in the incomprehensibility principle of Fraser (Fraser, 2013) (cf. Sect. 3).

Case (ii): When $\mathcal{M}$ does not depend explicitly on q, then ($\partial \mathcal{M}$ /∂q)=0 and eq.(13a) gives

$$\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{M}}{\partial \dot{q}}\right)+\frac{\partial}{\partial \mu}\left(\frac{\partial \mathcal{M}}{\partial \overline{q}}\right)=0$$

or $$\frac{\partial}{\partial t}\left[\frac{\partial \mathcal{M}}{\partial \dot{q}}+\bar{t}\,\frac{\partial \mathcal{M}}{\partial \overline{q}}\right]=0 \quad or \quad \left(\frac{\partial \mathcal{M}}{\partial \dot{q}}+\bar{t}\,\frac{\partial \mathcal{M}}{\partial \overline{q}}\right)\equiv P_t = constt. \text{ (with respect to t)}, \quad (15)$$

also $$\frac{\partial}{\partial \mu}\left[\frac{1}{\bar{t}}\frac{\partial \mathcal{M}}{\partial \dot{q}}+\frac{\partial \mathcal{M}}{\partial \overline{q}}\right]=0 \quad or \quad \frac{1}{\bar{t}}\frac{\partial \mathcal{M}}{\partial \dot{q}}+\frac{\partial \mathcal{M}}{\partial \overline{q}}\equiv P_\mu = constt.\ (with\ respect to\ \mu).$$

We call P_μ as the 'conjugate c-momentum'. In this case it turns out to be a conserved quantity provided $(\partial\bar{t}/\partial t)\simeq 0$, or, when the variation of experience-based time with event-based time is negligibly small.

Construction of the cenergy function $\mathcal{M}$: As mentioned before, for a functional human Being, the function $\mathcal{M}$ consists of two parts: 'gross part' ($\mathcal{M}_g$), an analogue of kinetic energy term, and 'subtle part' ($\mathcal{M}_s$), an analogue of potential energy term. Naturally, the gross part $\mathcal{M}_g$ is needed for the immediate use in daily life (corresponds to a fast and immediate efforts) and depends explicitly on $\dot{q},\overline{q},\bar{t}$ whereas the subtle part $\mathcal{M}_s$ is needed for the 'second thought' (corresponds to slow and later efforts) and in general could be a function of $q,\dot{q},\overline{q},\bar{t},t,\mu$ and is denoted by $V(q,\dot{q},\overline{q},\bar{t},t,\mu)$. Here, however, we restrict to the case when the potential energy analogue is V(q,t,μ)— a function of the variables in the q-t-μ space. As a matter of fact at subtle level the roles of $\dot{q},\overline{q}, and\ \bar{t}$ are very much there in taking decisions and using memory but it is internally consistent with the self and the same occurs at such a fast pace that it goes unnoticed in the outer world. Thus, in analogy with the energy function of classical mechanics, we can postulate a structure of $\mathcal{M}(=\mathcal{M}_g+\mathcal{M}_s)$ as

$$\mathcal{M}=a\,\dot{q}^2+b\overline{q}^2+c\,\bar{t}^2+V(q,t,\mu)\,. \qquad (16)$$

Here, a, b and c are suitable dimensional parameters adjusted keeping in mind that the function $\mathcal{M}$ restores the dimensions of energy. Further note that the potential term V here can be positive or negative depending on the attitude of the individual. In fact, if a person follows the systematic of positive pursuit with respect to the world-order then the V-term in (16) is positive; otherwise this term can be considered as negative. For such a choice of $\mathcal{M}$ eqs.(13) yield the following psycho-dynamical equations for the evolution of the system:

$$a\ddot{q}+b\overline{\overline{q}}-\frac{1}{2}\frac{\partial V}{\partial q}=0 \quad ; \quad c\,\bar{\bar{t}}-\frac{1}{2}\frac{\partial V}{\partial t}=0\,, \qquad (17a,b)$$

where $\ddot{q}=(\partial^2 q/\partial t^2)$, $\overline{\overline{q}}=(\partial^2 q/\partial\mu^2)$, $\bar{\bar{t}}=(\partial^2 t/\partial\mu^2)$.

It may be mentioned that unlike the space time mediated dynamical variables in physical theories, the variables $q,\dot{q},\overline{q},t,\bar{t},\mu$ here are not ideally isolated or independent; they rather do have inter-dependence. This is mainly because of the plurality character of consciousness which immediately after the switching on, becomes all-pervading and also the creator of space and time. This sets the limitation on the use of eqs.(17a,b).

A GUIDE FOR THE APPLICATIONS OF EQS. (17) OR (13)

As far as the applications of psycho-dynamical eqs.(17) (or for that matter of their generalized version (13)) is concerned, individual's personality appear in the applications. Besides the way in which an individual is involved in the consciousness-manifesting phenomena his attitude or psychology towards the world- or cosmic-order also matters and accordingly the role of 'potential term' in $\mathcal{M}$ is considered. As how one reacts in the circumstances created by his own consciousness with regard to emotions, thinking and/or memory, is highly subjective and as such it is not easy to quantify. Still one can attempt to solve eqs.(17) for some typical cases, may be under some simplifying assumptions.

A word of caution regarding the use of eq.(17) is necessary. In fact here q is considered as a simple function of variables t and μ. But in reality it is not so, when one plans a task at a given point in space his consciousness constantly undergoes through all sorts of time and/or consciousness channels as a part of decisions. As a result, q becomes a complicated functional of functions of t and μ , namely q=q(u(t), v(μ)) even at the initial stages of planning. Such considerations will however help in setting the boundary conditions for the solution of (17) or of (13). In fact, in this case the boundary conditions need to be fixed in accordance with initial and final stages (or mindsets) of the task.

Even for the choice of V as V=V(q,t,μ) in (16), the solution of eqs.(17) is not easy. However, we demonstrate the viability or effectiveness of these results for some representative cases. Note that the role of the potential term V in (17) disappears in either of the cases, namely when (i) V(q,t,μ)=0, or (ii) V(q,t,μ)=V(μ). In these cases eqs.(17) take the form

$$a\ddot{q}+b\overline{\overline{q}}=0 \quad , \qquad c\overline{\overline{t}}=0 . \qquad (18a, b)$$

Eq. (18b) admits the solution

$$t(\mu)=d(\mu-\mu_0), \qquad (19)$$

where d is a constant and $\mu=\mu_0$ at t = 0. Note that (19) is the solution of (18b) for the ideal case when V=0; otherwise for a realistic situation, eqs.(17a,b) should be solved for the forms

$$a\ddot{q}+b\overline{\overline{q}}-W_1(t,\mu)=0 \quad , \qquad c\overline{\overline{t}}-\frac{1}{2}W_2(\mu)=0, \qquad (20a, b)$$

which, in general, will admit very complicated solution depending on the forms of V or of W_1 *and* W_2. We restrict ourselves from going into further details here. For the solution of the partial differential eq. (18a) we discuss the method of the separation of variables (see, for example, Pipes and Harvill, 1970) in Appendix A by considering it as an analogue of Laplace equation in two dimensions.

In Appendix A, the solutions of eq.(17) are discussed corresponding to the cases when the subtle component V(q,t,μ) in $\mathcal{M}$ is either absent or has mild dependence on its arguments. These solutions can however offer a basis for developing the perturbation solution of the problem, if the necessity arises for certain forms of V(q,t,μ).

From the applications point of view, one can notice some relevance of the solution (A11) of eq.(17). Note that under the assumption of separability of the solution of q(t,μ) for t and μ-variables, the solution

$$q_m(t,\mu) = B_m \ \exp(-m\pi\mu(a/b)^{1/2}/\tau) \quad \sin\left(\frac{m\pi\, t}{\tau}\right), \qquad \text{(A11)}$$

describes the space variation with μ and t in a particular mode and mood (mindset) m. Further, note that the time and consciousness themselves adjust in such a way that the space q_m in this particular mode is optimum. For a car driver in a crowded lane or for a person crossing a crowded road such different mindsets arise and finally result in to the net path q(t,μ) (cf. eq.(A12)).

CONCLUDING DISCUSSION

This article is an attempt to give still deeper foundations to the subject of psycho-science in rigorous mathematical terms, particularly with reference to the phenomena belonging to world-2 (cf. Sect.1). After all it is a necessity of modern times when other soft disciplines like sociology and economics are trying to marry with physics to give birth to the subjects of socio-physics (see, for example, (Galam, 2012)) and econo-physics (see, for example, (Doyane Farmer, et. al, 2005)), then why psychology should be left untouched from this kind of development. Although further applications of the psycho-dynamical equations derived (cf..eqs. (13) and (17)) here are due , but some demonstrated in Sect.5, will pave new ways to understand the human behavior and personality better.

With a view to accounting for the consciousness in physical theories or to understand the role of mental processes in physical phenomena, several ways and methods have been proposed in the past (see, for example, Penrose, 1994; Kaushal, 2009; Stapp, 2009; Mlodinow, 2012; Bruneberg et. al, 2018; Samuel et. al, 2019) in different contexts. Susie Vrobel (2011) discusses several such phenomena and tries to understand them through fractal time. An important concept about time advanced in the work of Vrobel is that of ‘length’ and ‘depth’ of the time interval. This concept seems to have a close connection with the concept of experience-based and event-based space time intervals discussed in Sect.3.

In recent years, Karl Friston and his group (see, for example, Bruneberg et. al, 2018; Samuel et. al, 2019, and the references therein) have also advocated a free energy (variation) principle to tackle the hard problem of consciousness related to a society. However, their variation method, based on Bayesian statistics, is different from the one used in the present work in the sense that psycho-mechanics here is investigated as a formal extension of the grand theory of classical mechanics. Akin to this, the author has also proposed earlier (Kaushal, 2009) an extended complex phase space approach for this purpose. A similar ‘effort integral’ is constructed there and the relevance of the corresponding equations of motion is discussed but in a complex phase space by introducing the concept of different imaginaries for the elements of the set and for the mapping on the set. An important application of this approach is demonstrated for the case of Chevreul hand-held pendulum (Kaushal, 2016). The approach followed here is however more elegant and perhaps easy to extend to the quantum domain.

Note that the derived psycho-dynamical equations (13) require further investigation with reference to different consciousness-manifesting phenomena belonging to world-2. An interesting feature one can note from the solution (A9) of eq.(18a) is the symmetry of the solution q(t,μ) with respect to the interchange of t and μ along with the change in the sign of constant, viz. $k^2 \to -k^2$. This perhaps is the outcome of the assumption of treating μ-variable at par with t-variable in the trio of space, time and consciousness.

We have dealt with the typical case of V=0 in Appendix A. The separability assumption for the solution of (18a) has further simplified the solution of the problem of zig-zag path for a person in a crowded lane. Note that the detailed features of the movement are still not clear from the solution (A9) or (A11). In reality one should look for the solution of (20a) and (20b) together for certain ansatz for the potential functions $W_1(t,\mu)$ and $W_2(\mu)$. Such studies are in progress.

Appendix A: Solution of Eq. (18a)

Here, we present the solution of eq.(18a) by writing it in the form (as an analogue of Laplace equation in two dimensions),

$$\frac{\partial^2 q}{\partial x^2}+\frac{\partial^2 q}{\partial y^2}=0 \ , \qquad \text{(A1)}$$

where $x=t/\sqrt{a}\,,\ y=\mu/\sqrt{b}$. Next, we choose a separable form of the solution (see, for example, (Pipes & Harvill, 1970)) of (A1), viz., $q(x,y)=u(x).v(y)$. Using this form of q, eq.(A1) can be written as

$$\frac{1}{u}\frac{d^2u}{dx^2}=-\frac{1}{v}\frac{d^2v}{d\mu^2} \ . \qquad \text{(A2)}$$

Note that the left hand side of this equation is a function of x alone, whereas the right hand side is of y alone. Hence each of the expression in (A2) must be independent of x and y and should be a constant (say, $-k^2$). Thus, (A2) is equivalent to a pair of equations of the form

$$\frac{d^2u}{dx^2}=-k^2u \quad , \quad \frac{d^2v}{dy^2}=k^2v \ , \qquad \text{(A3, A4)}$$

whose solutions can be written immediately as (Pipes & Harvill, 1970)

$$u(x)=A_1\cos kx+A_2\sin k\,x \qquad \text{(A5)}$$

$$v(y)=A_3\,e^{ky}+A_4\,e^{-ky} \ . \qquad \text{(A6)}$$

Thus,

$$q(x,y)=e^{-ky}(B_1\cos kx+B_2\sin kx)+e^{ky}(B_3\cos kx+B_4\sin kx), \qquad \text{(A7)}$$

or $$q(t,\mu)=e^{-k\mu/\sqrt{b}}(B_1\cos\frac{kt}{\sqrt{a}}+B_2\sin\frac{kt}{\sqrt{a}})+e^{k\mu/\sqrt{b}}(B_3\cos\frac{kt}{\sqrt{a}}+B_4\sin\frac{kt}{\sqrt{a}}), \qquad \text{(A8)}$$

where $A_i's$ and $B_i's$ (for i=1, …, 4) are the arbitrary constants to be fixed after using the boundary conditions.

For choosing the boundary conditions recall the nature of variable μ. Ideally speaking, when μ=∞, the roles not only of q and t but also of $u(t\sqrt{a})$ and $v(\mu\sqrt{b})$ disappear. Hence in (A8) we should have $B_3=B_4=0$. On the other hand, we also have q=0 for t=0. This implies that we cannot have a cosine term in the solution and hence $B_1=0$. Thus, an acceptable solution turns of to be

$$q(t,\mu)=B_2\,e^{-k\mu/\sqrt{b}}\ \sin(kt/\sqrt{a}\,),, \qquad \text{(A9)}$$

which, for the form (19), automatically satisfies, q=0 at $\mu=\mu_0$, otherwise for an arbitrary t(μ) as the solution of (20b), one should have q=0 at t=τ (say) and $\mu=\mu_0$ and thus requiring from eq.(A9) that

$$\sin k\,\tau/\sqrt{a}=0 \ . \qquad \text{(A10)}$$

This suggests the values of k as $k = \frac{m\pi\sqrt{a}}{\tau}$, where m=0,1,2… .. Finally, the solution of (18a) takes the form

$$q_m(t,\mu) = B_m \ e^{-m\pi\mu\sqrt{a/b}/\tau} \ \sin\left(\frac{m\pi t}{\tau}\right), \qquad \text{(A11)}$$

where B_m is an arbitrary constant for given m. In fact m accounts for different modes that cover the net path q(t,μ), now given by

$$q(t,\mu) = \sum_{m=0}^{\infty} q_m(t,\mu) \ . \qquad \text{(A12)}$$

Note that in a realistic situation *q(t,μ),* as the solution of eqs. (20), is not separable in *t* and μ.

Appendix B: Further Remarks on the Experience-and Event-Based Space Time Intervals

If one goes with the concept of Vrobel (Vrobel, 2011) of 'time density' in terms of 'the length' and 'the depth' or of 'the succession' and 'the now' of a time interval Δt and uses a similar concept for the space interval Δq as well in terms of 'the succession of the steps (or space slices)' and 'the here', then there is a possibility to obtain a new type of space time transformation for the happenings in the world-2 and the world-1 (cf. Sect.1). This transformation, no doubt, has to be a nonlinear (logarithmic) one (unlike the Galilean or the Lorentz transformation in physics which connects the events in two moving reference frames) but also has roots in the Weber-Fechner law. We demonstrate here a new type of kinetics that emerges out of these concepts and is relevant for further application of eqs.(13) or (17).

As mentioned in the text $\dot{q}, \bar{q}, \bar{t}$ in (16) are not independent; they are rather connected in view of the fact that q=q(t,μ) and t=t(μ). This, in turn, implies that

$$dq = \frac{\partial q}{\partial t} dt + \frac{\partial q}{\partial \mu} d\mu$$

or

$$v(\mu) \equiv \frac{dq}{dt} = v_{ph} + v_d \qquad \text{(B1)}$$

where $v_{ph} = (\partial q/\partial t)$, is the physical velocity that corresponds to the world-3, and $v_d = (\bar{q}/\bar{t})$, is the additional derived velocity that emerges out of consciousness-dependence in the system. Note that the contribution of v_d-term in the (B1) could be positive or negative depending on the pleasant or unpleasant experience by an individual. Although v_d is not fix but there is an upper bound on v_{ph} due to the velocity of light c, i.e., $v_{ph} \le c$, which implies that

$$v(\mu) \le c + v_d \ . \qquad \text{(B2)}$$

Weather it is the case of pleasant and unpleasant experiences or of the functioning of conscious and sub-conscious minds towards an experience, in either situation, the same v(μ) plays a dual role depending on the magnitude of the ratio $(\bar{q}/\bar{t})$ or of v_d. In fact, for the experiences of the conscious domain $\mathcal{M}_g$, $\bar{q}$ *and* $\bar{t}$ and hence the ratio $(\bar{q}/\bar{t})$ have mild (or at times hardly any) dependence on μ whereas in the sub-conscious domain, $\mathcal{M}_s$, $\bar{q}$, $\bar{t}$ and also the ratio $(\bar{q}/\bar{t})$ have strong dependence on μ.

Alternatively, one can derive v(μ) in a crude way using Weber-Fechner law (5) written for space and time intervals, viz.,

$$\Delta q_{ex} = k_1 \ln(\Delta q_{ev} / A_1), \; \Delta t_{ex} = k_2 \ln(\Delta t_{ev} / A_2).$$

Obviously, k_1 *and* k_2 here are the dimensional constants of proportionality and also are the functions of µ. Thus, we have

$$v(\mu) = \frac{\Delta q_{ex}}{\Delta t_{ex}} = \frac{k_1(\mu)}{k_2(\mu)} \cdot \frac{\ln(\Delta q_{ev} / A_1)}{\ln(\Delta t_{ev} / A_2)}. \qquad \text{(B3)}$$

Here, (k_1 / k_2) while defines the 'velocity' v_d, the remaining factor can be linked with v_{ph} but in a complicated way, may be under some simplifying assumptions regarding Δq_{ev} *and / or* Δt_{ev}.

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Conflict of Interest

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