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Critical Review

Mathematics teaching learning strategy for mathematical reasoning: a critical review of past studies

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ABSTRACT

Mathematics education is very important for students during their schooling, as it makes the students think logically to find logical solution of the problem. An appropriate programme for developing mathematical reasoning may help the students to become aware of the statistics, shapes, patterns that surround them in their day to day life. Mathematical reasoning skill may be useful to the students in the fields of science, engineering and finance too. Observing the manifold usability of mathematical reasoning in day to day life, it was felt to ascertain some patterns which can guide us for making an effective mathematical reasoning programme for the students. With this major objective in mind, this paper has been an attempt to critically review the past researches. The fifteen research studies related to the teaching of mathematics are included in the review. An attempt has been made to review the past researches keeping in mind the major points—the problem of study, tools, sample, data analysis and result. Result of research studies shows that techniques used for experimental group were found to be as effective as traditional method or more effective than the traditional ones. The analysis of past studies confirms the positive response from the students and teachers towards the methodology in developing mathematical reasoning programme by different researchers.

Keywords: *Mathematics, Mathematics teaching, Mathematical reasoning*

Roger Bacon (1214-1294) an English philosopher, scientist and scholar of the 13th century, once stated:
"Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of the world."

Today's era is a science and technology era. Mathematics is the basis of the development of science and technology, so mathematics is called the queen of science. Knowledge of mathematics depends on the ability of mathematical reasoning, so it is imperative for students to develop mathematical skill from primary level. To improve mathematical skill in the students, there has been change in teaching learning process.

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Mathematical reasoning is a skill that enables students to use all other mathematical skill. Using mathematical reasoning, students can understand how to assess the situation, choose strategies to solve problems, come to logical conclusions, describes solutions and how to apply solutions. If the students develop mathematical reasoning, they will become an independent mathematical thinker.

Mathematics experts divide mathematics into two parts. First is the conceptual knowledge and second one is the procedural knowledge. Therefore, the subject recommended at any point of time in mathematics education will have to establish integration between conceptual and procedural knowledge (Hiebert and Carpenter, 1992). The mathematics researchers associated with national council of teachers of mathematics (1991) have categorically established that what the student learns is fundamentally linked with how he learns. Thus, teaching design plays an important part in teaching and learning mathematics.

BACKGROUND AND FRAMEWORK

In this section presented discussion on the difference between learning of competency and learning via competency. This is important because the purpose of the review is to determine the extent to which problem solving and reasoning were examined and which abilities were used as a learning tool.

According to Niss (2003) Mathematical competency is defined as "the ability to understand, judge, do, and use mathematics in intra- and extra-mathematical contexts and situations where mathematics play or could play role." Thus, mathematical competency is a component of general mathematical aptitude, specific skills such as problem solving or mathematical reasoning. In the present analysis, it is analyzed whether the teaching design achieves the goals through problem solving or/ and reasoning.

An education design must have a learning goal that can direct the design (National Research council, 2000) but to reach that goal, there must be an idea, a claim, how to reach the goal (Van den Acker, 2010). To characterize the teaching design found in the retrieved articles, the review uses a characteristic version of Van den Acker's (2010) teaching design modified by Lithner (2017). Lithner's edition uses goals, claims and arguments, which convey information about:

1. The goal to be achieved through suggested teaching design.
2. Claim how to reach a set goal.
3. Theoretical and empirical arguments supporting claims about reaching the set goal.

METHODOLOGY

The present review included fifteen research studies related to the teaching of mathematics, of which nine were researched in India and six in abroad. All selected research studies are experimental type.

Table 1: The terms search for literature review

Term-1	Term -2
Reasoning	Method
Logic	Design
Mathematics	Programme
Teach	Instruction

Teaching can have different educational goals, for which different teaching schemes can be used. Different claims can be made about how to reach the goals of teaching and different arguments can be presented to support the claims made. The purpose of the literary review is to show the educational research of mathematics to solve the problem of education and / or to characterize the education connected with logic. In this present review research study reviewed on the bases of problem of study, tools, data analysis, result and teaching design. The presented review mainly based on the following questions.

1. What was the research goal?
2. Which were the tools used for the research?
3. What method was used for the data analysis in research?
4. What were the findings of research and what type of teaching design was use for achieve goals and claims?

Goals of study

For each article, the goals, claims, and theoretical and empirical arguments for the formation of education were identified by closely reading the full text of each article. To know the goal of the design, the following analytical question arose: "What goals are to be achieved through the suggested learning or intervention?" The answer to the question was considered when the purpose of the design intervention was identified.

Tools

The tools are selected by the researchers keeping in view the research objectives. The choice of tools remains important for the achievement of research objectives. The tools used for the research selected for the present review can be divided into two types depending on the composition as shown in the table below.

Table 2: Type of tools and researcher

Tool	Researcher
Self made	Kundu (2008), Okigbo and Osuafor (2008), Stephan (2015), Janier and others (2008), Rawal (2010), Ali and others (2010), Vijayan (2015),
Standardized and Self made	Meera (2007), Patel (2009), Shah (2011), Bincy (2016), Sing (2017), Kalsia (2018), Lee (2018), Berenger (2017)

As shown in the table-2, seven researchers used self- made tools while seven researchers used both standardized and self-made tools. As standardized tool I.Q. test designed by Dr. K.G. Desai, IQ test developed by Dr. Jyoti Dave, mathematical creativity test (Sharma, Yogesh & Sansonwal; 2011), mathematical discomfort criteria (Sharma, Yogesh, & Sansonwal; 2011) were used. In addition, the researchers used self-made devices for a variety of purposes. Including pre-test, achievement test, CAI package, mathematical laboratory based designed lesson plans, multimedia courseware, worksheets, creative teaching program, multimedia package, module based on concept mapping, inventory-based mathematics themed achievement test, geometric argumentation test included.

Data analysis

After reviewing the research, it was found that in previous researches the average, standard deviation, ANOVA, ANCOVA, T test, F-ratio, X^2 test were used to analyze and interpret the information.

RESULT

Result of research demonstrates that techniques used for experimental group were as effective as traditional method or more effective than the traditional ones. The Students and teachers gave positive response for the programme and methodology used by the researchers.

Table 3: Articles included in the full-text literature review

Article	Goal	Claim	
		Overarching idea	Via PS or reasoning
1	PS	SS	PS
2	Other competency	TU	PS, Reasoning
3	Reasoning	TS, SS	Reasoning
4	Reasoning	TS, SS	Reasoning
5	Other competency	TS, SS	PS
6	Other competency	TS	PS
7	Reasoning	TS	Reasoning
8	Reasoning	TU	Reasoning
9	Other competency	TS, SS	Reasoning
10	Other competency	EM	Reasoning
11	PS	SS,	PS
12	Other competency	TU	Reasoning
13	Other competency	TS	Reasoning
14	Other competency	EM	Reasoning
15	Other competency	TS, SS	Reasoning
Total: 26	PS: 2 Reasoning: 4 Other competency: 9	SS: 2 TS: 4 SS & TS: 5 EM: 2 TU: 2	PS: 4 Reasoning: 10 Reasoning & PS: 1

Abbreviations: PS – Problem solving, TS – Teacher-led scaffolding, SS – Students' self-assisted scaffolding, EM – Emergent models, TU – Teaching unit

This section first presents a table -3 listing the reviewed studies, showing the various goals and claims, when analyzing goals and claims of study as well as use of reasoning and problem solving to reach set goals is shown. The numbers shown in the first column in Table 3 refer to the study given within the Appendix. To illustrate the results, instead of presenting a complete analysis of all the reviewed articles, examples of the reviewed articles are cited.

Goals as part of a teaching design

According to Ball & Bass (2003) two central goals of mathematics education are to support students' development of the interconnected key competencies problem solving and reasoning. Out of the 15 studies selected, two had problem solving, four had reasoning and nine had other competency type goals. An example of a study with a goal of problem-solving related learning was one by Ali R. et.al. (2010) in which researcher presented result of Effect of Problem-Solving Method in Teaching Mathematics on the Achievement of Mathematics Students of standard- 8. A study presented by Lee (2018) is example of study with goal, in which argumentative activities were used as pedagogical tool to help grade 5 students to classify geometric shapes by their properties. The study presented by Priti kalsia

(2018) is example of study with other competency type goal, in which the goal was to study the effect of inquiry-based learning approach on achievement and attitude towards mathematics of IX graders in relation to their mathematical anxiety.

Claims as part of teaching design

Analysis of the research reveals that there are four important ideas of origin claims. These broad ideas are classified into subsequent subsections.

1. Teacher-led scaffolding
2. Student self-assisted scaffolding
3. Emergent models and
4. Teaching units.

The analysis shows that similar claims may be linked to different goals. For example, both goals associated with problem solving and goals associated with other capabilities can be claimed to be reached by some kind of scaffolding. Claims about teacher-led scaffolding and student self-help scaffolding were mostly linked to goals about problem solving or reasoning, while claims about the Emergency Model were linked to goals about other capabilities. The important idea of the education units was linked to goals about both problem solving and other skills. In the following subsections, four broad ideas of claims are presented.

Teacher-led scaffolding

The first explicit discussion of a claim relating to reaching design intervention goals through a teacher-led scaffolding "process that enables a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts" Wood, Bruner and Ross (1976). Lee (2018) reported on the study with two main goals of teaching students (I) learn how to solve mathematical problems, (II) learn together. Researcher constituted scaffolding strategy in the context of the whole class in which social dimensions promoting metacognitive skills and group study were promoted. Studies confirm that learning trough group study strategies have a positive effect on students' ability to solve math problems.

Student self-assisted scaffolding

The second idea of claims related to reaching design intervention goals by Teach students how to use scaffolding strategies or solution method schemes to solve their problem. Ali R.et al.(2010) claimed in a study that the goal of scaffolding students has been achieved through scheme focused on understanding and planning. The claim was supported by experimental results, which indicated that the students participating in the experimental group developed a better problem-solving strategy than the reference group. However, the authors said that the intervention may be a little shorter for students to internalize these strategies.

Emergent models

The third idea of claims related to reaching design intervention goals by emergent models. According to Gravemeijer (1999) the essence of emergency modeling is to allow students to take a situation-specific problem and model it informally instead of parting. Gradually, with the support of the teacher, the model develops into more formal formal mathematics. In this way, students gain new mathematical knowledge and understanding. For example, Stephan (2015) claimed on an education design intervention aimed at allowing students to learn meaningfully addition and subtraction.

Teaching units

The fourth idea of claim pertained to the various teaching units in which the teachers implemented certain types of detailed instructions. This important idea was seen in various studies but did not include claims linked to teacher-led scaffolding, student self-help scaffolding, or models of emergence. Berenger (2017) presented a design study aimed at overcoming the difficulties of the understanding of 2-dimensional shapes for middle year students. It was claimed that this could be done by providing teachers with instructional resources to generate autonomy and support students in decision making. Berenger (2017) was quite detailed about how the teaching sequence should be carried out.

CONCLUSION

The techniques used in the reviewed research to teach mathematics like advanced organizer model, computer-assisted instruction programme, laboratory use in mathematics, multimedia courseware, mathematics and chess integrated training, creative teaching programme, problem solving methods, activity-based model and coordination model and inquiry-based learning approach were found to be as effective as traditional method or more effective than the traditional ones. Most of the research studies provided positive feedback on the programme and methodology used by the researchers, in future, further research could be undertaken to develop new programmes and methods to make mathematics education more effective. The scope of such research can also be extended in higher education. Special emphasis should be given to the work on programmes for developing mathematical reasoning. Researchers and teachers can be encouraged to carry out research work on mathematics teaching and learning.

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Conflict of Interest
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APPENDIX

The following articles were included in the review. The numbers refer to the article numbers in table 3.

1. Ali R. et.al. (2010). *Effect of Problem Solving Method in Teaching Mathematics on the Achivement of Mathematics Students*. Reasearch Paper, www.ccsenet.org/ass
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